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Model Predictive Control and Control Barrier Function for Motion Planning with Obstacle Avoidance

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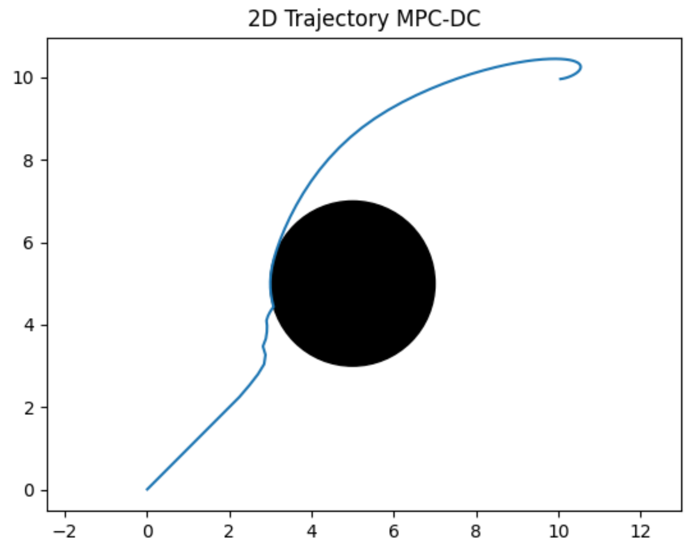
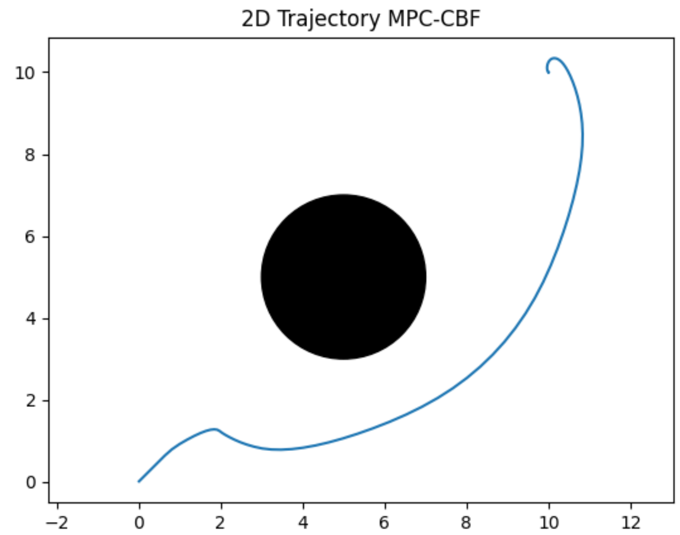
Problem Formulation

- 2D double integrator model.
- Safely navigate (avoid collision with obstacle) from start to goal state while satisfying constraints (actuation limits).
- Both static and moving obstacles.
- Comparison of 3 three different algorithms.





Control Barrier Function

- Define set $C \subset D \subset R^n$.
- Impose constraints to link control input to safety constraints.
- For a control affine system:

$$\dot{x} = f(x) + g(x)u$$

where α is an extended class K infinity function.

$$\sup_{u \in U} [L_f h(x) + L_g h(x)u] \geq -\alpha(h(x)).$$

$$\begin{aligned} \mathcal{C} &= \{x \in D \subset \mathbb{R}^n : h(x) \geq 0\}, \\ \partial\mathcal{C} &= \{x \in D \subset \mathbb{R}^n : h(x) = 0\}, \\ \mathrm{Int}(\mathcal{C}) &= \{x \in D \subset \mathbb{R}^n : h(x) \cancel{>} 0\}. \end{aligned}$$

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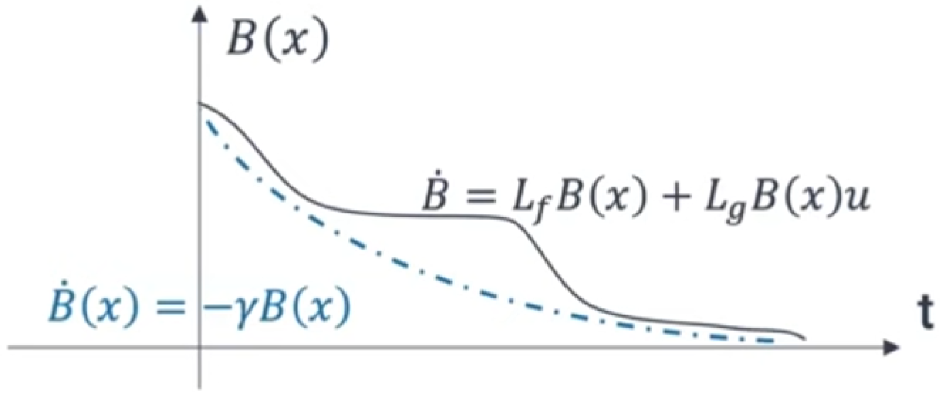

Credit to "Jason Choi -- Introduction to Control Lyapunov Functions and Control Barrier Functions", YouTube

Algorithms

- Method 1: MPC-DC
- Method 2: MPC-DC
- Method 3: CBF-QP

MPC-CBF

$$
\begin{aligned}
\min_{u_{k:N-1}} \quad & \sum_{k=0}^{N-1} \left((x_{goal,k} - x_k)^T Q (x_{goal,k} - x_k) + u_k^T R u_k \right) \\
\text{s.t.} \quad & x_{k+1} = A_d x_k + B_d u_k, \quad k = 0, \dots, N-1, \\
& L_f h(x_k) + L_g h(x_k) u \geq -\alpha(h(x)) \quad k = 0, \dots, N-1 \\
& u_{min} \leq u_k \leq u_{max}, \quad k = 0, \dots, N-1, \\
& x_0 = x_{start}, \\
& x_N = x_{goal},
\end{aligned}
$$

where $L_f h(x_k)$ is the *Lie* derivative of the function $h(x)$ along the vector field f, $L_g h(x_k)$ is the *Lie* derivative of the function $h(x)$ along the vector field g, and α is an extended class K_∞ function.

MPC-DC

$$
\begin{aligned}
\min_{u_{k:N-1}} \quad & \sum_{k=0}^{N-1} \left((x_{goal,k} - x_k)^T Q (x_{goal,k} - x_k) + u_k^T R u_k \right) \\
\text{s.t.} \quad & x_{k+1} = A_d x_k + B_d u_k, \quad k = 0, \dots, N-1, \\
& g(x_k) \geq 0, \quad k = 0, \dots, N-1 \\
& u_{min} \leq u_{t+k|t} \leq u_{max}, \quad k = 0, \dots, N-1, \\
& x_0 = x_{start}, \\
& x_N = x_{goal},
\end{aligned}
$$

where $g(x_k)$ is the function that describe the euclidean distance between the robot and the obstacle.

CBF-QP

$$
\begin{aligned}
u(x) = \underset{u \in \mathbb{R}^m}{\operatorname{argmin}} \quad & \frac{1}{2} \| u - k(x) \|^2 \\
\text{s.t.} \quad & L_f h(x) + L_g h(x) u \geq -\alpha(h(x))
\end{aligned}
$$

where $k(x)$ is provided from a nominal controller.

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Thank you !

Any questions ?